UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 3rd FEBRUARY 2005

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

SOLUTIONS LEAFLET

This solutions leaflet for the IMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** In increasing order, the numbers are 4.004, 4.04, 4.044, 4.4, 4.44.

2. **D** 10% of one million is 100 000; 10% of one thousand is 100; 100 000 – 100 = 99 900.

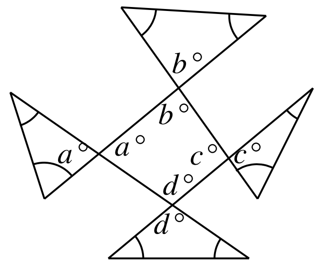
3. **A** As the hands move anticlockwise, at 1:30 pm they would have the appearance they would normally have at 10:30. So, as the clock is viewed in a mirror, it would have the appearance shown in A.

4. **B** The values of the expressions are, respectively, 202, 2005, 9010, 27 217 and 65 026.

5. **C** If a number is divisible by 6 then it must also be divisible by 2, so is even. ‘The sum of any two odd numbers is even’ is also true. However, not all multiples of 9 are odd (e.g. 18) and the sum of any two even numbers is even rather than odd, so two of the four statements are true.

6. D The offer gives the purchaser two items for the price of one and a half items. So the average cost per item is the same as four items for the price of three.

7. B Let the sizes of the interior angles of the quadrilateral in the centre of the figure be $a°$, $b°$, $c°$ and $d°$. Then $a + b + c + d = 360$. Each of these angles is vertically opposite to one of the unmarked angles in the four triangles, as shown. So the sum of the marked angles plus the four angles of the quadrilateral is equal to the sum of the interior angles of four triangles, that is 720°. Hence the sum of the marked angles is 720° − 360° = 360°.



8. A The required fraction $= \dfrac{6\frac{3}{4}}{24} = \dfrac{27}{96} = \dfrac{9}{32}$.

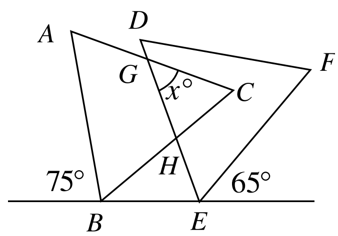
9. D The area of triangle A $= \frac{1}{2} \times 2 \times 3 = 3$; the area of parallelogram B $= 1 \times 3 = 3$; the area of triangle C $= \frac{1}{2} \times 3 \times 2 = 3$; the area of rectangle E $= 1 \times 3 = 3$. However, the area of triangle D $= 3 \times 3 - (3 + 3 + \frac{1}{2}) = 2\frac{1}{2}$.

10. C Let the weights in kg of the head and body of the fish be h and b respectively. Then $h = 9 + \frac{1}{3}b$ and $b = h + 9$. So $b = 9 + \frac{1}{3}b + 9$, that is $\frac{2}{3}b = 18$, which gives $b = 27$. Hence $h = 18$, so the whole fish weighed 54kg.

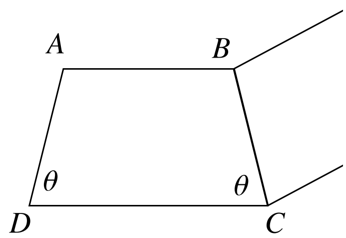
11. C The side of length 5cm cannot be the hypotenuse of the right-angled triangle as it is shorter than the side of length 6cm. If the 6cm side is the hypotenuse, then the third side of the triangle has length $\sqrt{11}$ cm. If the 6cm side is not the hypotenuse, then the hypotenuse has length $\sqrt{61}$ cm. These are the only two possibilities.

12. B Ten gallons of honey would provide enough fuel for one bee to fly about 70 000 000 miles. So the number of bees which could fly 1000 miles is approximately 70 000 000 ÷ 1000, that is 70 000.

13. D Considering the angles at B and E:
$\angle CBE = (180 - 75 - 60)° = 45°$;
$\angle DEB = (180 - 65 - 60)° = 55°$. Therefore
$\angle GHB = (45 + 55)° = 100°$ (exterior angle theorem) and, using the same theorem, $\angle HGC = (100 - 60)° = 40°$.

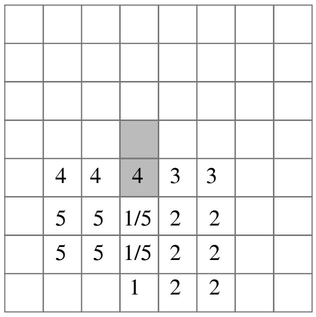


14. C Let $ABCD$ be the cross-section of one of the stones, as shown. As $AD = BC$, $ABCD$ is an isosceles trapezium with $\angle ADC = \angle BCD$ (the proof of which is left to the reader). If $\angle ADC = \angle BCD = \theta$, then 2θ is the interior angle of a regular 20-sided polygon, namely $(180 - 360/20)°$, which equals 162°. So θ is 81°.



15. A Four bags of porridge contain one-fifth of a bag of wheatbran. So the proportion of wheatbran in the porridge is 1/20, that is 5%.

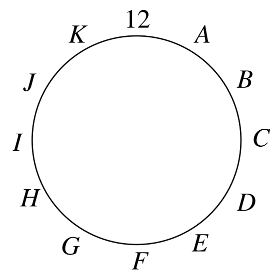
16. B The two dark squares are those on which the block stands originally. After the first move, it occupies the squares labelled '1' or '1/5'. After the second move it occupies the squares labelled '2' and so on. After the fifth and final move, it occupies the squares labelled '5' or '1/5'.
As can be seen, 19 squares are occupied altogether.



17. C The volume of 1kg of platinum is $(1000/21.45)\text{cm}^3$, that is approximately 50cm^3. So 1 tonne of platinum has a volume of approximately $50\,000\text{cm}^3$, which is $1/20\text{m}^3$. The volume of platinum produced per year is therefore about 5m^3 and the total volume of platinum ever produced is approximately 250m^3. This is the volume of a cuboid measuring $10\text{m} \times 5\text{m} \times 5\text{m}$, which is comparable to a house.

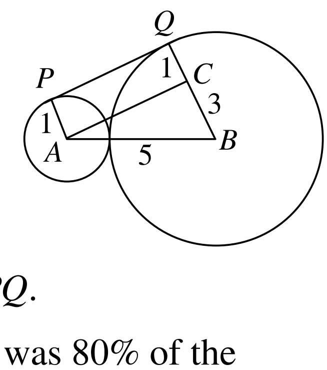
18. E The area of the rectangle is 48cm^2, so the unshaded area is 12cm^2. Therefore $\frac{1}{2} \times x \times 2 + \frac{1}{2} \times (6 - x) \times 8 = 12$, that is $x + 24 - 4x = 12$, so $x = 4$.

19. C For ease of reference, label the points $A, B, C, \ldots, K$ as shown. First note that $A = 3$ and $K = 9$ or vice versa. With no loss of generality, let $A = 3$. Then the only possible values for B, C and D are 7, 8 and 2 respectively. This gives $E = 1$ or 4 and, as $K = 9$, $J = 1$ or 6. If $J = 1$, then $E = 4$ and the only remaining possibilities for I, H and G are 5, 10 and 11 respectively. This means that $F = 6$, but $11 + 6$ is not a triangle number, so J is not 1 and must, therefore, be 6. This means that $I = 4$ and hence $E = 1$. The remaining values may now be assigned: $H = 11, G = 10$ and $F = 5$.



20. D The mean of the nine consecutive positive integers is the fifth of the numbers, so their sum is nine times the fifth number. As nine is itself a perfect square, the sum will be a perfect square if and only if the fifth number is a perfect square. For the options given, the fifth numbers are 114, 124, 134, 144 and 154 respectively.

21. E The diagram shows points A and B, which are the centres of the two circles, and C, the point on BQ such that AC is parallel to PQ. As radii PA and QB are both perpendicular to tangent PQ, $APQC$ is a rectangle. So $\angle ACB$ is a right angle. The length of $AB = 1 + 4 = 5$; the length of $BC = 4 - 1 = 3$. So, by Pythagoras' Theorem, $AC = \sqrt{5^2 - 3^2} = 4$, which, therefore, is also the length of PQ.



22. A Let the number of cases solved in 2003 be x. Then, as this was 80% of the number of cases, there were $5x/4$ cases to solve in 2003. So the number of cases to solve in 2004 was $5x/4 \times 6/5$, which is $3x/2$. Inspector Remorse solved 60% of these cases, that is $3x/2 \times 3/5$, which is $9x/10$. So the change in the number of cases solved was a 10% decrease.

23. E The equations of the three lines must be considered in pairs to find the coordinates of their points of intersection, i.e. the coordinates of the vertices of the triangle. It is left to the reader to show that these are $(-15,\ -9)$, $(0,\ 6)$ and $(5,\ 1)$. The area of the triangle may now be found by enclosing it in a rectangle measuring 20×15 and subtracting the areas of the three surrounding triangles from that of the rectangle. This gives $300 - (112\frac{1}{2} + 12\frac{1}{2} + 100) = 75$.

24. A Figure (i) shows a net of the cube on which a possible path has been drawn, while figure (ii) shows a diagram of the cube on which the same path has been drawn. Each edge of the network which joins a corner to a face centre has length $1/\sqrt{2}$, while each edge which joins two adjacent corners has length 1. So the length of the path shown is $1 + 12 \times 1/\sqrt{2}$, that is $1 + 6\sqrt{2}$. This is the length of the shortest path along the edges of the network which passes through all 14 vertices. To prove this, we first note that to connect the 14 vertices we need a minimum of 13 edges, so the length of the shortest path must be at least $13 \times 1/\sqrt{2}$. A path of this length would move alternately between corners and face centres, but as there are 8 corners and 6 face centres this is impossible. At least one of the edges on the shortest path, therefore, must join two corners. So the length of the shortest path must be at least $1 + 12 \times 1/\sqrt{2}$. The diagrams show that such a path does exist so we are able to conclude that $1 + 6\sqrt{2}$ is indeed the length of the shortest path.

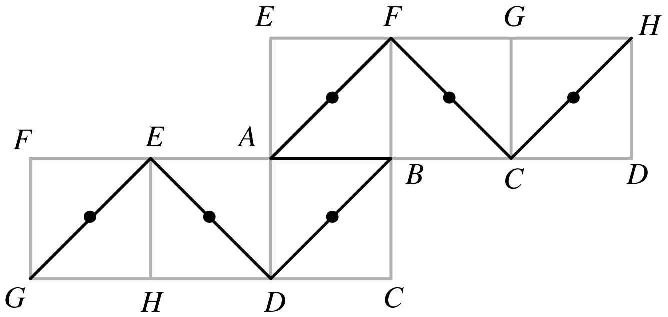


Figure (i)

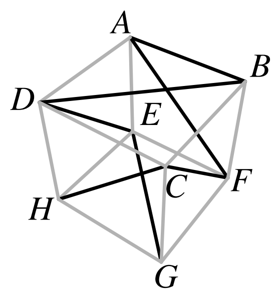
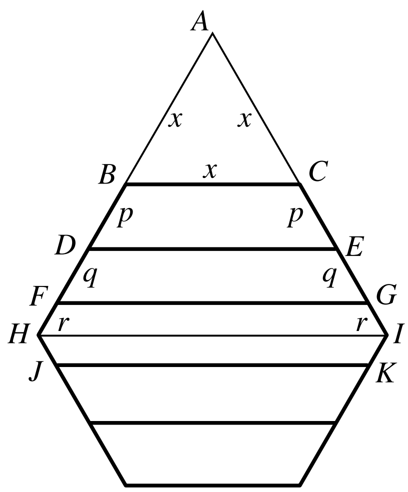


Figure (ii)

25. B Each exterior angle of a regular hexagon $= 360° \div 6 = 60°$, so when sides HB and IC are produced to meet at A, an equilateral triangle, ABC, is created. Let the sides of this triangle be of length x. As BC, DE and FG are all parallel, triangles ABC, ADE and AFG are all equilateral. So $DE = DA = p + x$; $FG = FA = q + p + x$.



The perimeter of trapezium $BCED = x + p + x + 2p = 2x + 3p$; the perimeter of trapezium $DEGF = (p + x) + (q + p + x) + 2q = 2x + 2p + 3q$; the perimeter of hexagon $FGIKJH = 2((q + p + x) + 2r) = 2x + 2p + 2q + 4r$.

So $2x + 3p = 2x + 2p + 3q$; hence $p = 3q$. Also $2x + 2p + 3q = 2x + 2p + 2q + 4r$; hence $q = 4r$. So $p : q : r = 12r : 4r : r = 12 : 4 : 1$.